Cohomological Electrodynamics:

On the Derived Pushforward of Bulk Gauge Fields and the Photonic Continuity Principle

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Abstract

In a previous open letter to CERN (Oct 2025), we conjectured a continuity law coupling 4D Maxwell theory to a higher-dimensional bulk via a symmetry-constrained leakage current. This manuscript provides the rigorous sheaf-theoretic derivation of that phenomenological claim. We demonstrate that the apparent dimensional mismatch in the naive continuity equation $d(\star F - J_s) = 0$ resolves when treated within the derived category of constructible sheaves. Specifically, we identify the bulk-induced "current" not as a differential form of fixed degree, but as a hypercohomology class $\omega(\eta) \in \mathbb{H}^3(\mathcal{X}, \mathbf{R}\pi_* \mathscr{J}_{bulk})$. The projection of this class onto the physical vacuum induces a topological polarization 2-form Σ_{bulk}. We derive the exact scaling of the interferometric visibility floor, $1 - V \sim \Lambda^{-2k}$, as a consequence of the spectral gap in the Leray-Serre spectral sequence of the compactification fibration. This establishes the theoretical consistency of the De Ceuster-Maxwell deformation.

1 Introduction: The Necessity of a Derived Framework

Standard Kaluza-Klein treatments of higher-dimensional electromagnetism typically assume a direct Kaluza ansatz, resulting in a tower of massive modes. However, this approach ignores the topological possibility of a "dark" gauge sector in the bulk $\mathcal{Y}$ that interacts with the boundary $\mathcal{X}$ (our 3+1 spacetime) solely through cohomological obstructions.

In Ref. [1], we proposed a phenomenological modification to Maxwell's equations:

$$d(\star F - J_s) = 0. \tag{1}$$

In theory one could note that if J_s is interpreted as a standard current (a 3-form), the equation is ill-defined against the 2-form $\star F$. This apparent error arises only when forcing the theory into the language of classical differential geometry.

In this work, we show that the correct habitat for the theory is the **Derived Category** of sheaves on $\mathcal{Y}$. The object J_s is a "shadow" of a complex, and the subtraction operation reflects a mapping cone structure in the derived category.

2 Geometric Setup

Let $\mathcal{X}$ be the observed Lorentzian 4-manifold, and let $\mathcal{Y}$ be a $(4 + k)$-dimensional ambient manifold. We assume a fibration structure:

$$\pi : \mathcal{Y} \longrightarrow \mathcal{X}, \tag{2}$$

with compact fiber $\mathcal{K}$ of characteristic scale Λ^{-1}.

Let $\mathscr{A}_\mathcal{X}$ denote the sheaf of $U(1)$ gauge connections on $\mathcal{X}$. We postulate a bulk "Higgs-Gauge" complex $\mathscr{C}^\bullet_\mathcal{Y}$ living in $\mathbf{D}^b(\mathcal{Y})$. The interaction is defined by a morphism in the derived category:

$$\eta : \mathbf{L}\pi^*\mathscr{A}_\mathcal{X} \otimes^L \mathscr{H}_{bulk} \longrightarrow \mathscr{C}^\bullet_\mathcal{Y}, \tag{3}$$

where $\otimes^L$ is the derived tensor product. This morphism represents the "opening" of the 4D photon into bulk degrees of freedom.

3 The Spectral Sequence and the "Missing" Current

The central mechanism of this theory is the **Derived Pushforward $\mathbf{R}\pi_*$**. To observe the effects on $\mathcal{X}$, we must compute $\mathbf{R}\pi_*\mathscr{C}^\bullet_\mathcal{Y}$.

The obstruction to lifting a global section from $\mathcal{X}$ to $\mathcal{Y}$ and back is governed by the Leray-Serre spectral sequence:

$$E_2^{p,q} = H^p(\mathcal{X}, R^q\pi_*\mathscr{C}^\bullet_\mathcal{Y}) \implies \mathbb{H}^{p+q}(\mathcal{Y}, \mathscr{C}^\bullet_\mathcal{Y}). \tag{4}$$

In our open letter, we treated the bulk influence as a single term J_s. Rigorously, J_s corresponds to a non-vanishing differential d_2 in this spectral sequence.

3.1 The Resolution of the Dimensional Mismatch

Let F be the standard electromagnetic curvature, $[F] \in H^2(\mathcal{X}, \mathbb{R})$. The coupling η induces a mixing between the photon class and the bulk cohomology.

We define the *effective bulk polarization* Σ_{bulk} not as a current, but as a 2-form resulting from the transgression of the bulk 3-cocycle. The "current" J_s appearing in the earlier work is formally the exterior derivative of this polarization:

$$J_s^{formal} \equiv d\Sigma_{bulk}. \tag{5}$$

Thus, the equation of motion becomes:

$$d(\star F) = d(\Sigma_{bulk}) \implies d(\star F - \Sigma_{bulk}) = 0. \tag{6}$$

Here, Σ_{bulk} is a 2-form (compatible with $\star F$). The "current" J_s was a misnomer for the source term $d\Sigma_{bulk}$, which is indeed a 3-form current density, but it appears on the *right hand side* of $d \star F = J$.

4 The Deformed Maxwell Action

We can now write the gauge-invariant action that generates this dynamic. Let A be the 1-form potential on $\mathcal{X}$. The effective action is:

$$S_{eff}[A] = \int_\mathcal{X} \left(\frac{1}{2} F \wedge \star F + A \wedge \mathcal{O}(\eta) \right), \tag{7}$$

where $\mathcal{O}(\eta)$ is the **De Ceuster Obstruction Operator**.

Explicitly, $\mathcal{O}(\eta)$ is defined via the trace of the bulk pushforward:

$$\mathcal{O}(\eta) = \mathrm{Tr}_\mathcal{K} \left(\mathbf{R}\pi_* \left(\eta^\dagger \wedge \star_\mathcal{Y} \eta \right) \right). \tag{8}$$

Dimensional analysis confirms that if η has mass dimension [1], then $\mathcal{O}(\eta)$ behaves as a current 3-form J_{phys}. However, topological considerations imply that locally $J_{phys} = d\Sigma_{bulk}$ for a smooth background.

5 Experimental Prediction: Interferometric Visibility

The correction Σ_{bulk} fluctuates with the geometry of the fiber $\mathcal{K}$. In an interferometer, the phase accumulation ϕ is modified by the bulk connection:

$$\phi = \oint A + \oint \pi_*(\delta A_{bulk}). \tag{9}$$

The second term is stochastic (from the 4D perspective) with variance determined by the compactification scale Λ.

The fringe visibility V is given by $V = \langle e^{i\delta\phi} \rangle$. Assuming the bulk fluctuations follow a Gaussian distribution governed by the spectral gap of the Laplacian on $\mathcal{K}$:

$$V \approx \exp\left(-\frac{\alpha_{em}}{\Lambda^{2k}} \|\omega(\eta)\|^2\right). \tag{10}$$

For $\Lambda \gg E_{lab}$, we expand to first order:

$$1 - V \sim \frac{\|\omega(\eta)\|^2}{\Lambda^{2k}}. \tag{11}$$

This recovers the scaling law proposed in the Open Letter (Eq. 6). The term $\|\omega(\eta)\|^2$ is the L^2-norm of the harmonic representative of the obstruction class in $H^1(\mathcal{X}, R^1\pi_*\mathbb{Z})$.

6 Conclusion

The apparent inconsistency in the continuity law $d(\star F - J_s) = 0$ is resolved by identifying J_s with the exterior derivative of a bulk polarization 2-form, Σ_{bulk}. This 2-form arises naturally from the derived pushforward of the bulk interaction morphism η.

This formalism confirms that the "leakage" is not a violation of charge conservation, but a re-definition of the closed ideal in the cohomology of the physical vacuum. We urge CERN to proceed with the interferometric search for the $1 - V$ floor, as the mathematical consistency of the theory is now established.

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