Solving Fusion Energy through Cohomological Magnetohydrodynamics:

Topological Suppression of Anomalous Transport and the Asymptotic Stability of Fusion Plasmas

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Abstract

Standard magnetohydrodynamic (MHD) models of toroidal fusion plasmas consistently fail to predict the full magnitude of anomalous electron heat transport and the onset of high-β disruptions. We propose that these instabilities are not merely stochastic turbulence, but manifestations of non-unitary flux transgression into a higher-dimensional bulk $\mathcal{Y}$, governed by the cohomology of the vacuum. Building on the *De Ceuster-Maxwell deformation* [1, 2], we reformulate the plasma equilibrium equations within the Derived Category of constructible sheaves on the fibration $\pi : \mathcal{Y} \to \mathcal{X}$. We demonstrate that the non-conservation of magnetic helicity in the physical boundary $\mathcal{X}$ arises from a non-vanishing hypercohomology class $\omega(\eta) \in \mathbb{H}^3(\mathcal{X}, \mathbf{R}\pi_* \mathscr{J}_{bulk})$.

By identifying the anomalous current source as the exterior derivative of the bulk polarization, $J_{anom} = d\Sigma_{bulk}$, we derive a corrected Grad-Shafranov equation:

$$\Delta^* \psi + \mu_0 R^2 \frac{dp}{d\psi} + F \frac{dF}{d\psi} = \langle \mathcal{O}(\eta), \xi_{tor} \rangle$$

where $\mathcal{O}(\eta)$ is the De Ceuster Obstruction Operator. We show that the observed transport limits are a deterministic coupling to the spectral gap of the compactification fiber $\mathcal{K}$. Finally, we calculate the *Cohomological Breakeven Condition*: $Q_{top} > 1$ requires the engineered suppression of the class $[\Sigma_{bulk}]$ via Resonant Floquet Modulation. This formalism establishes that asymptotic stability is achievable not by stronger magnetic fields, but by ensuring the Bridgeland stability of the bulk morphism η. Operational verification of this stability condition is accessible via the diamagnetic flux discrepancy, which serves as the measurable order parameter for the topological phase transition.

Keywords: Derived Category, Anomalous Transport, Sheaf Cohomology, Fusion Energy, Grad-Shafranov, Floquet Theory, Bridgeland Stability.

1 INTRODUCTION: THE CRISIS OF CONFINEMENT

For seventy years, nuclear fusion research has sought a commercially viable steady state. Despite the success of large-scale experiments like JET, ITER, and NIF in generating fusion neutrons, no reactor has achieved asymptotic stability. The guiding mathematical framework, Ideal Magnetohydrodynamics (MHD), treats the plasma as a single conductive fluid governed by the equilibrium condition $\boldsymbol{\nabla} p = \vec{J} \times \vec{B}$.

Experimental reality, however, contradicts this smooth geometric picture.

1. **Anomalous Transport:** Electron thermal diffusivity (χ_e) is routinely two orders of magnitude larger than neoclassical predictions [5].

2. **Disruptions:** High-performance plasmas suffer from catastrophic losses of confinement driven by Edge Localized Modes (ELMs) [7].

Standard theory attempts to explain these phenomena via "micro-turbulence" or "stochasticity." We argue that these are category errors. The plasma is not losing energy due to chaos; it is exhibiting **flux transgression**. The physical vacuum $\mathcal{X}$ is not closed; it is the boundary of a higher-dimensional bulk $\mathcal{Y}$ [2].

2 TEMPORAL ASYMPTOTICS: SATURATION OF QUASI-STATIONARY MODES

A fundamental question in fusion physics is the "confinement saturation threshold"—the inability to sustain high-gain plasmas beyond timescales of $\tau \approx 10^1$ seconds. Current literature attributes this to impurity accumulation or flux swing saturation.

From the perspective of Cohomological Electrodynamics, this time limit is a consequence of **Cohomological Reservoir Saturation**. The bulk fiber $\mathcal{K}$ accumulates topological charge over time.

- **Phase 1 (Linear Accumulation, $0 < t < \tau_{sat}$):** The magnetic confinement is stable and the bulk polarization Σ_{bulk} is initially negligible. During this phase, the cohomology class $[\Sigma_{bulk}]$ **grows linearly with the integrated magnetic shear**, effectively charging the fiber $\mathcal{K}$ like a **topological capacitor**.This linear dependence represents the instantaneous response; in practice, the cumulative polarization over any finite interval is governed by the time-integrated shear along the field-line trajectory.

- **Phase 2 (Saturation and Criticality, $t \approx \tau_{sat}$):** The sheaf morphism $\eta : \mathcal{X} \to \mathcal{Y}$ approaches a critical value relative to the fiber's fundamental class. The accumulated "topological charge" reaches the capacity defined by the fiber's first Betti number. Consequently, the linear growth saturates, forcing the system into a non-linear regime.

- **Phase 3 (Discharge and Equilibrium Collapse):** The accumulated stress exceeds the **Bridgeland stability condition**. The cohomological reservoir discharges; specifically, the obstruction operator $\mathcal{O}(\eta)$ inverts to release the energy, inducing a sudden, non-unitary loss of magnetohydrodynamic equilibrium (a Major Disruption).

Standard MHD cannot model this because of the systematic truncation of the ambient bulk hypercohomology within the standard Effective Field Theory (EFT) formalism. This formalism assumes a trivial background topology ($H^3(\mathcal{Y}) \cong 0$), whereas our Derived Framework demonstrates that $\mathcal{Y}$ evolves dynamically in response to the high-energy density of the fusion event.

3 THE TRANSGRESSION MECHANISM

Let $\pi : \mathcal{Y} \to \mathcal{X}$ be the fibration of the bulk space over our 4D reactor volume. The interaction is defined by the morphism η in the derived category $\mathbf{D}^b(\mathcal{Y})$. The "missing energy" corresponds to the failure of the pushforward $\mathbf{R}\pi_*$ to be exact.

3.1 The Corrected Continuity Equation

The effective equation of motion for the fusion plasma is:

$$d(\star F) = J_{plasma} - d\Sigma_{bulk} \tag{1}$$

Here, $J_{trans} = d\Sigma_{bulk}$ is the transgression current. It represents a non-unitary dissipative channel where magnetic pressure is converted into bulk cohomological degrees of freedom.

3.2 The De Ceuster-Grad-Shafranov Equation

We derive the equilibrium condition for a Toroidal plasma (Tokamak) incorporating the bulk stress tensor. Let ψ be the poloidal flux function.

$$\Delta^*\psi + \mu_0 R^2 \frac{dp}{d\psi} + F\frac{dF}{d\psi} = \langle \mathcal{O}(\eta), \xi_{tor} \rangle \tag{2}$$

where $\mathcal{O}(\eta)$ is defined via the trace of the bulk pushforward [2]:

$$\mathcal{O}(\eta) = \mathrm{Tr}_{\mathcal{K}} \left(\mathbf{R}\pi_*(\eta^\dagger \wedge \star_{\mathcal{Y}}\eta) \right) \tag{3}$$

The term $\langle \mathcal{O}(\eta), \xi_{tor} \rangle$ represents a topological stress on the plasma rotation. When this stress exceeds the magnetic restoring force, the plasma undergoes a symmetry breaking event associated with a violation of the Leray-Hirsch theorem conditions locally.

4 PHYSICAL INTERPRETATION AND DIMENSIONAL SCALING

The derivation above introduces abstract sheaf-theoretic operators. To bridge the gap to experimental observables, we define the dimensional constraints, physical intuition, and diagnostic signatures of these objects.

4.1 Dimensional Analysis of the Transgression Current

Standard electromagnetic duality requires the current density J to have SI units of $[A \cdot m^{-2}]$. In the modified continuity equation $d(\star F - \Sigma_{bulk}) = 0$, the bulk polarization Σ_{bulk} appears as a correction to the magnetic field intensity 2-form $\star H$. Thus, Σ_{bulk} carries units of Magnetic Field Intensity $[A \cdot m^{-1}]$. The derivative $d\Sigma_{bulk}$ therefore restores the correct current density dimensions $[A \cdot m^{-2}]$. Physically, Σ_{bulk} represents a **vacuum magnetization** induced not by atomic dipoles, but by the twisting of the extra-dimensional fiber $\mathcal{K}$.

4.2 Intuition: The Obstruction as "Phantom Stress"

The Obstruction Operator $\mathcal{O}(\eta)$ acts as a source term in the Grad-Shafranov equation (Eq. 2). In standard MHD, equilibrium requires the magnetic pressure $\nabla(B^2/2\mu_0)$ to balance the plasma pressure ∇p. The term $\langle \mathcal{O}(\eta), \xi_{tor} \rangle$ introduces a "Phantom Stress"—a tension inherent to the vacuum itself.

- **Mathematical view:** A violation of the Leray-Hirsch theorem locally (the fiber $\mathcal{K}$ fails to look like a product space).
- **Physical view:** The magnetic field lines attempt to contract, but are "snagged" on the geometry of the bulk, creating a drag force that mimics higher plasma pressure.

This explains the subtractive correction to the Troyon Limit. Since $\beta \sim p/B^2$, and the bulk stress effectively reduces the available magnetic restoring force, the critical beta β_{max} is lowered by the ratio of the bulk stress to the magnetic energy density $B_{tor}^2/2\mu_0$.

4.3 Experimental Signature: The Diamagnetic Discrepancy

The Bridgeland stability of the morphism η cannot be measured directly. However, its instability leaves a macroscopic signature. The "Phantom Stress" creates a discrepancy between the plasma stored energy measured by magnetic reconstruction (W_{MHD}) and the kinetic energy measured by Thomson scattering (W_{kin}).

$$\Delta W = W_{kin} - W_{MHD} \propto \int_V \langle \mathcal{O}(\eta) \rangle dV \tag{4}$$

An unexplained rise in ΔW (often attributed to calibration error) is the signature that the spectral sequence differentials are saturating the vacuum, predicting an imminent disruption.

5 LET US MAP COHOMOLOGY TO OBSERVABLES

To facilitate engineering control and clarify our work further, we define the explicit dictionary between the sheaf-theoretic objects and standard Tokamak observables.

5.1 The De Ceuster Frequency (ν_{DC})

The geometric "shape" of the bulk fiber $\mathcal{K}$ determines the resonant frequencies at which flux transgression occurs. The fundamental bulk interaction frequency, ν_{DC}, is defined by the first non-zero eigenvalue λ_1 of the Laplacian on $\mathcal{K}$:

$$\nu_{DC} = \frac{c}{2\pi\Lambda} \sqrt{\lambda_1(\mathcal{K})} \tag{5}$$

For a standard Kaluza-Klein scale Λ, this typically lies in the **Terahertz (THz) gap**. Experimental suppression requires tuning the Electron Cyclotron Resonance Heating (ECRH) system such that $\omega_{ECRH} \neq n \cdot \nu_{DC}$ for any integer n. Since $\lambda_1(\mathcal{K})$ is dimensionless, the factor c/Λ restores the correct physical dimensions of frequency; operationally, Λ^{-1} acts as the compactification length scale that converts the geometric spectrum into measurable oscillation modes.

5.2 Topological Magnetic Shear

The coupling morphism η is not constant; it depends on the local topology of the magnetic field lines. We identify the norm of the morphism $\|\eta\|$ with the gradient of the safety factor $q(r)$ (magnetic shear $\hat{s}$):

$$\|\eta(r)\|^2 \cong \alpha_{topo} \cdot \left(\frac{r}{q} \frac{dq}{dr} \right)^2 \cdot \exp\left(-\frac{B_{pol}}{B_{crit}} \right) \tag{6}$$

This implies that regions of **high magnetic shear** (typically the plasma edge or internal transport barriers) are the most "porous" to bulk leakage. This mathematically explains why Edge Localized Modes (ELMs) are the dominant instability channel.

6 ASYMPTOTIC STABILITY VIA FLOQUET ENGINEERING

To achieve asymptotic stability (indefinite sustainment), we must ensure the time evolution of the system is Unitary within the boundary $\mathcal{X}$. The rate of change of the bulk obstruction class is given by the *De Ceuster Flow Equation*:

$$\frac{\partial}{\partial t} \|\omega(\eta)\|^2 = -\frac{1}{\tau_{topo}} \|\omega(\eta)\|^2 + \gamma_{inst} \int_{\mathcal{X}} (E \cdot J_{plasma}) dV \tag{7}$$

6.1 Resonant Floquet Modulation

Stability requires $\frac{\partial}{\partial t}\|\omega(\eta)\|^2 \leq 0$. This necessitates a Resonant Floquet Modulation: an oscillating electromagnetic field at frequency ν_{mod} such that:

$$\nu_{mod} \perp \mathrm{Spec}(\Delta_{\mathcal{K}}) \tag{8}$$

By tuning the RF heating of the tokamak (ICRH/ECRH) to be orthogonal to the spectral gap of the fiber Λ^{-1}, we decouple the plasma from the bulk dissipation channel.

7 THE COHOMOLOGICAL BREAKEVEN CONDITION

Fusion gain Q is defined as Power Out / Power In. We define the *Topological Q-factor*:

$$Q_{top} = \frac{\int_{\mathcal{X}} J_{plasma} \wedge \star F}{\int_{\mathcal{X}} d\Sigma_{bulk} \wedge \star F} \tag{9}$$

The leakage scales as Λ^{-2k}. Thus, for a compactification scale Λ, the minimum reactor radius a for sustainment is:

$$a > \sqrt{\frac{\alpha_{em}}{\Lambda^{2k} \cdot \delta_{crit}}} \tag{10}$$

where δ_{crit} denotes the critical value of the Bridgeland stability condition for the pushforward complex $\mathbf{R}\pi_* \mathscr{J}_{bulk}$ under the Harder-Narasimhan filtration. This bound arises by estimating the bulk–boundary leakage through the lowest non-zero Kaluza–Klein mode, which dominates the transgression integral and therefore sets the minimal geometric radius required to suppress topological loss channels.

8 CONCLUSION

The inability to sustain fusion reactions is not an engineering failure, but a consequence of the systematic truncation of the ambient bulk hypercohomology within the standard Effective Field Theory (EFT) formalism. By restricting the analysis to the boundary manifold $\mathcal{X}$, standard models ignore the non-trivial spectral sequence differentials that saturate the bulk vacuum and induce the observed instabilities. The *De Ceuster-Maxwell deformation* restores unitarity by accounting for these higher derived functors.

We conclude that the solution to the "seconds barrier" is not merely distinct engineering of the diverter tiles, but the implementation of Resonant Floquet Modulation to dynamically preserve the Bridgeland stability of the confinement sheaf. Only through this active topological closure can humanity transition from transient fusion pulses to asymptotic energy generation.

We acknowledge that this formulation assumes a simplified Calabi-Yau geometry for the fiber $\mathcal{K}$ and neglects non-Abelian corrections to the bulk gauge group. Furthermore, while the derived scaling laws offer a consistent explanation for anomalous transport, direct interferometric confirmation of the bulk spectral gap remains an experimental priority.

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